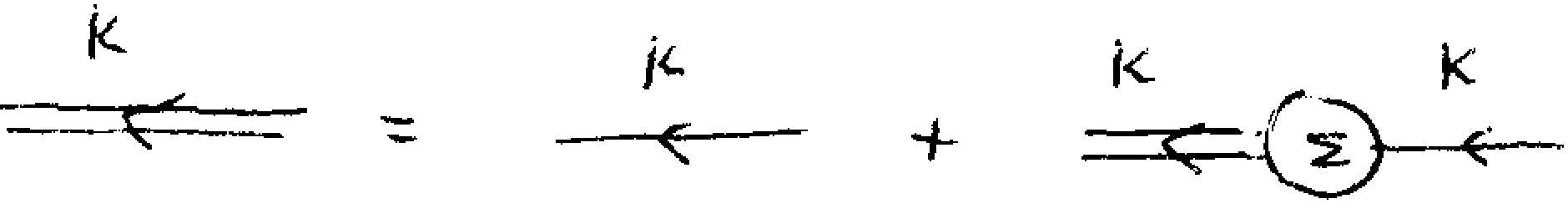
**Normal Metal Excitations**

**Electron Self-Energy**

The motion of the ions doesn’t affect that of the electrons too much, especially because they do effectively screen the ions motion, reducing the ionic potential to a short ranged potential, which effectively makes the electron-ion interaction weak. So we would only need to include the effects of the e-e interaction on the electron propagator.



and,



But yeah, I guess for the most part the result here would just reduce to those already explored in the context of the pure e-e interaction. Mahan does say that the self-energy correction does give the electrons within a Debye energy of EF a rather large effective mass. The size of this correction in various metals is correlated with whether and how strongly they will superconduct.

**Example**

Let’s just write out what the first term above is. Going to use the renormalized e-phonon vertex, and full (up to RPA) phonon GF.



Let’s see what g2 is, for future reference,



Now we can use the equation we worked out in the Electron-Phonon folder/Interaction file,



So we have:



Okay, and now going to plug this into the lowest order phonon self-energy diagram,

Diagram, schematic

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And in symbols,



(the ωq is implicitly that of the longitudinal branch – see Interaction file) And we could do the Matsubara sum with the contour integral approach, but I’ll not. Maybe I will. So using that identity (see Stat Mech/Math Appendix)



we can do the sum over frequencies, to get:



So we have:



Let’s note the following:



So can say,



Could specialize to T = 0. Then nB(ωq) = 0,



Well the retarded self-energy is:



Gonna take the real part. And change to spherical integration variables. Will use εk-q = εk + εq – kqcosθ/m, and already doing the dφ integral → 2π.



The first integral is:



Don’t really want to keep going there. Let’s take a look at the other one,



We’ll have to do IBP. First note,



So we have:



where in last line we use fact that at T = 0, n´F(ε) = -δ(ε). What does the first one look like?



Let’s work out the consequence of nF on the bounds of integration,



The first inequality is irrelevant since q must always be > 0, and k - √2mμ is < 0 I think. And the other,



Again, we can ignore the second inequality. So can say,



Don’t really want to proceed further with that either. And I’m told that the really important one is the remaining one, though I don’t know how we’d know…



Can change variables q → -q+k



where the surface integral is over the Fermi surface, and vq = q/m (might recall Free Day/Electrons/Excitations/Properties file). One thing we can see already is that when ω = 0, then the whole term is zero. Now ω = 0 gives the location of the Fermi surface. And so if this correction is zero, then the location of the Fermi surface is unchanged. Can also see that the correction is small if ω is large, as then ln|(ω-ωk-q)/(ω+ωk-q)| → ln|ω/ω| → 0. Let’s look at ω << ωD, and k close to the Fermi surface. Then since q is *on* the Fermi surface ωk-q > ω *for the most part* (but remember it ωk-q can only range between 0 and ωD as illustrated in the Interaction file).

A picture containing diagram

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And so we can say,



Further, for a spherical Fermi surface, we can say vq depends only on the magnitude, which means we can say vq = vF. And filling in g, we have:



Could try to evaluate this, but have to remember that ωk-q ranges between 0 and ωD and if |k-q| > qD, then it goes back to ωk-q-G, where G is reciprocal lattice vector (see Interaction file). I guess we can fill in g,



Then we have:



Not going to try to actually evaluate this though. Let an estimate suffice. So we’ll replace ωk-q with ωD. And since k and q are around the Fermi surface, we’ll just say,



And since qTF ~ kF (see Free Day/Electrons/Thermal Equilibrium folder), we can say,



And I’ll treat the surface as spherical, so:



Now a0 ~ 1/kF, so we can set this to 1. And then we have:



Okay, so our self-energy equation becomes:



So we see a renormalization of the effective mass,



Obviously ΩE/ωD is quite substantial (or can be). Or if you go back up in the derivation, we can see that the correction is, more generally, proportional to the electron density of states ρF = 2∫d3q/(2π)3; so we’d expect the correction to be large where ρF is large. And as we’ll recall from the Free Day/electrons/excitations/Fermi surfaces file, transition metals have a large density of states, typically, near the Fermi surface. So we’d expect these metals to have large effective masses. The large phonon contribution to the electron’s effective mass is ascribed to the ‘polarization’ cloud that the electron has to drag with it everywhere it goes. The polarization cloud is the local deformation of the lattice that the electron induces as it attracts the neighboring ions to itself. It turns out that this correction is most substantial for metals that superconduct, as this is kind of a precursor to the overdamping phenomenon whereby the lattice polarization induces an effective attractive interaction between electrons (well, see Interaction file). Copying a picture below from Ashcroft + Mermin of the correction.

A picture containing text, envelope

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